Interference Alignment Schemes for MIMO Channels

Gaurav Gupta and Mayank Bhardwaj

***Abstract*—In a $K-$user MIMO Interference channel the maximum degrees of freedom can be achieved using Interference Alignment technique [1] and the achieved degrees of freedom are exactly half of the signal space. To achieve interference alignment in MIMO systems a closed form solution is known only for 3 transmitters. Several modifications to this closed form solution exists which emphasize on sum rate maximization. For more than 3 transmitters iterative algorithms exist. In this report we will go through a wide range of schemes achieve interference alignment.**

***Index Terms*—multiple-input multiple-output (MIMO), multiuser, downlink, sum rate, interference alignment, degrees of freedom (dof)**

I. INTRODUCTION

Interference Alignment (IA) techniques aims at aligning the Interferences at the receiver such that they occupy half the signal space, leaving rest half for desired signal. It is shown in [1] that this technique achieves maximum dof in Interference channels. To achieve IA the precoding matrices are designed such that interference will align at the receivers occupying half of the signal space. The receiver processing is then performed at the particular receiver to separate the desired signal from interference. One popular way to do this is projecting the received signal onto orthogonal space of interference called zero-forcing, this way a complete nullification of interference is performed. Other ways are minimizing the mean squared error known as Minimum mean squared error (MMSE) approach.

To achieve interference alignment in single antenna systems, symbol extension id performed to increase the dimension of the transmitted signal by the base station [1]. In this report however, we will be concerned with the Multiple Input Multiple Output (MIMO) systems only. MIMO systems employing multiple antennas at both transmitter and receiver has already signal space of dimension greater than one. Therefore, the only problem remains is the design of precoding matrices to achieve IA at the receiver. The closed form solution for achieving IA is known only for $K \leq 3$ [1]. The optimality of this solution is itself a question as the authors of [1] have talked nothing about the sum rate performance of this solution.

The authors of [2] have provided the modification of this closed form solution and have designed the precoders keeping in mind, the sum rate maximization. They have also designed optimal receivers like modifications of Zero-forcing and MMSE receivers with the aim of sum rate maximization.

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For $K > 3$, two iterative algorithms exists to achieve IA [3]. The first algorithm is based on interference leakage power minimization while the second one is based on Signal to interference-plus-noise (SINR) maximization to achieve better sum rate than first algorithm. The main problem with these algorithms is their convergence which becomes a great issue if K increases.

The IA scheme does not talk about serving more than one users in each cell. However, practical scenarios should serve more than one users in each cell to improve spectral efficiency of the system. Hence, the extension of IA is Interfering Broadcast (IFBC) system. In this system, at each receiver there is inter-cell interference as well as inter-user interference. The authors of [4] proposed a grouping method for designing receiver and transmitter matrices to completely eliminate inter-user and inter-cell interference. This scheme is designed only for $2-$cell scenario and hence the authors of [5] extended the grouping method for any value of cells and users in each cell. The extension groups the users of next cell cyclically.

In this report we will go through all the above schemes to give an insight in the development of IA schemes from the beginning till now. We will discuss the algorithms and try to reproduce the results using simulations.

II. SYSTEM MODEL

We consider a $L-$user MIMO interference channel where the lth transmitter and receiver are equipped with M antennas. Fig. 1 shows how IA is achieved in $L = 4$ scenario. It can be seen from the figure that the interference is aligned in a space. The transmitted signal for the L cell system can be written as

$$\mathbf{y}^{[l]} = \sum_{k=1}^{L} \mathbf{H}^{[l,k]}\mathbf{x}^{[k]} + \mathbf{n}^{[l]} \tag{1}$$

where $\mathbf{n}^{[l]}$ is zero mean unit variance circularly symmetric additive white Gaussian noise vector (AWGN) at receiver l, $\mathbf{x}^{[k]}$ is $M \times 1$ signal transmitted by transmitter k, $\mathbf{H}^{[l,k]}$ is the matrix of channel coefficients between transmitter k and receiver l. The channel is assumed to flat fading with each entries of channel matrix are identically and independently (i.i.d.) distributed complex gaussian circluar symmetric random variables having unity variance. The transmit power at transmitter k is $E[||\mathbf{x}^{[k]}||^2 = P^{[k]}$.

We will now consider $L = 3$ scenario and see the closed form solution for achieving IA. For achieving IA we have to maximize the overlap between the interference received from neighbouring transmitters. Lets assume that the signal is precoded at the transmitter$-l$ with matrix $\mathbf{V}^{[l]}$ such that the

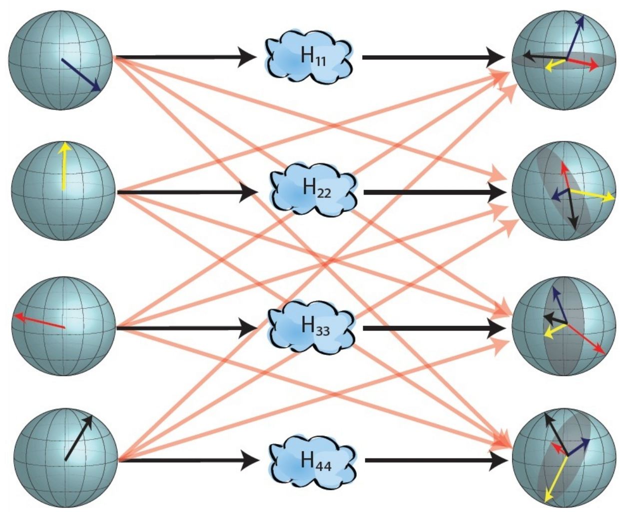

Fig. 1. IA system when $L = 4$

transmitted signal can be written as $\mathbf{x}^{[l]} = \mathbf{V}^{[l]}\mathbf{s}^{[l]}$, where $\mathbf{s}^{[l]}$ is $d^{[l]} \times 1$ symbol vector. The received signal at the receiver l can hence be written as

$$\mathbf{y}^{[l]} = \mathbf{H}^{[1,1]}\mathbf{V}^{[1]}\mathbf{s}^{[1]} + \mathbf{H}^{[1,2]}\mathbf{V}^{[2]}\mathbf{s}^{[2]} + \mathbf{H}^{[1,3]}\mathbf{V}^{[3]}\mathbf{s}^{[3]} + \mathbf{n}^{[1]} \tag{2}$$

To perfectly align the interference at each the receiver the following conditions must be satisfied [2]

$$\begin{aligned} \text{span}(\mathbf{H}^{[1,2]}\mathbf{V}^{[2]}) &= \text{span}(\mathbf{H}^{[1,3]}\mathbf{V}^{[3]}) \\ \text{span}(\mathbf{H}^{[2,1]}\mathbf{V}^{[1]}) &= \text{span}(\mathbf{H}^{[2,3]}\mathbf{V}^{[3]}) \\ \text{span}(\mathbf{H}^{[3,1]}\mathbf{V}^{[1]}) &= \text{span}(\mathbf{H}^{[3,2]}\mathbf{V}^{[2]}) \end{aligned} \tag{3}$$

where span($\mathbf{A}$) refers to the subspace spanned by the vectors of matrix $\mathbf{A}$. The conditions in (3) can equivalently be written as

$$\begin{aligned} &\text{span}(\mathbf{H}^{[1,2]}\mathbf{V}^{[2]}) = \text{span}(\mathbf{H}^{[1,3]}\mathbf{V}^{[3]}) \\ &\mathbf{H}^{[2,1]}\mathbf{V}^{[1]} = \mathbf{H}^{[2,3]}\mathbf{V}^{[3]} \quad \mathbf{H}^{[3,1]}\mathbf{V}^{[1]} = \mathbf{H}^{[3,2]}\mathbf{V}^{[2]} \end{aligned} \tag{4}$$

which again can be written as

$$\begin{aligned} &\text{span}(\mathbf{V}^{[1]}) = \text{span}(\mathbf{E}\mathbf{V}^{[1]}) \\ &\mathbf{V}^{[2]} = \mathbf{H}^{[3,2]\,-1}\mathbf{H}^{[3,1]}\mathbf{V}^{[1]}, \mathbf{V}^{[3]} = \mathbf{H}^{[2,3]\,-1}\mathbf{H}^{[2,1]}\mathbf{V}^{[1]} \end{aligned} \tag{5}$$

where $\mathbf{E} = \mathbf{H}^{[3,1]\,-1}\mathbf{H}^{[3,2]}\mathbf{H}^{[1,2]\,-1}\mathbf{H}^{[1,3]}\mathbf{H}^{[2,3]\,-1}\mathbf{H}^{[2,1]}$ and we can set $\mathbf{V}^{[1]}$ as

$$\mathbf{V}^{[1]} = [\mathbf{e}_1, ..., \mathbf{e}_{M/2}] \tag{6}$$

where $\mathbf{e}_1, ..., \mathbf{e}_{M/2}$ are eigenvectors of $\mathbf{E}$ and $\mathbf{V}^{[1]}$ is chosen like this because eigen space is transformation invariant. So we have a closed form solution to achieve IA when $L = 3$. We will now look upon its modifications for sum rate maximization.

III. Precoder Optimization

Before we address the precoder optimization let us introduce the notion of combination matrix first. Any matrix $\mathbf{A}$ can be decomposed as $\mathbf{A} = \mathbf{O}(\mathbf{A})\mathbf{C}(\mathbf{A})$, where $\mathbf{O}$ is is defined as a matrix which consists of the orthonormal basis vectors that span the column space of $\mathbf{A}$ and $\mathbf{C}(\mathbf{A})$ denotes the combination matrix of $\mathbf{A}$.

In some sense, the conventional IA only deals with determination of $\mathbf{O}(\mathbf{A})$ and does not worry about $\mathbf{C}(\mathbf{A})$. The combination matrix $\mathbf{C}(\mathbf{A})$ randomly chosen from the IA method does not affect the dof. However, this leads to a degradation of the sum rate performance [2]. We will now see that how this idea of combination matrix can be utilized to achieve sum rate gains and IA both. The conventional IA takes care of only interference. However, for having sum rate gains the rough orthogonality between desired signal space and interference signal space is essential. Therefore, the authors of [2] have proposed that instead of just selecting the first $M/2$ eigenvectors for determining $\mathbf{V}^{[1]}$ we will choose a $M/2$ subset of M eigenvectors such that the desired signal space as close orthogonal to interference as possible. For comparing orthogonality the metric used is chordal distance [6]. The orthogonality between the subspaces is proportional to the chordal distance, which implies that if chordal distance is more than the orthogonality between the subspaces is more.

1) Optimization using Zero-forcing beamforming: the QR decomposition of $\mathbf{V}^{[l]}$ obtained from the IA method as

$$\mathbf{V}^{[l]} = \mathbf{Q}^{[l]}\mathbf{R}^{[l]}, \qquad for l = 1, 2, 3 \tag{7}$$

where $\mathbf{Q}^{[l]}$ is an $M \times \frac{M}{2}$ matrix whose columns form an orthonormal basis for $\mathbf{V}^{[l]}$ and $\mathbf{R}^{[l]}$ is an $\frac{M}{2} \times \frac{M}{2}$ upper triangular matrix. The modified precoder can be written as

$$\mathbf{V}_{\text{zf}}^{[l]} = \mathbf{Q}^{[l]}\mathbf{C}_{\text{zf}}^{[l]} \tag{8}$$

where $\mathbf{C}_{\text{zf}}^{[l]}$ represents an $\frac{M}{2} \times \frac{M}{2}$ square matrix which satisfies the transmit power constraint $\text{Tr}(\mathbf{C}_{\text{zf}}^{[l]\,H}\mathbf{C}_{\text{zf}}^{[l]}) \leq P$. The modified precoding matrices will satisfy (3) and also will not affect the chordal distance [2] as

$$\text{span}(\mathbf{H}^{[i,j]}\mathbf{Q}^{[j]}) = \text{span}(\mathbf{H}^{[i,j]}\mathbf{Q}^{[j]}\mathbf{C}_{\text{zf}}^{[l]}) \tag{9}$$

Now we will apply the precoder in (8) to (1) and define the effective channel $\mathbf{H}_{\text{eff}}^{[i,j]}$ as $\mathbf{H}_{\text{eff}}^{[i,j]} = \mathbf{H}^{[i,j]}\mathbf{Q}^{[j]}$. Here all the matrices $\mathbf{H}_{\text{eff}}^{[i,j]}$ for $j = 1, 2, 3$ and $j \neq i$ spans the same space due to the interference aligning processing. Thus, we can choose one of the matrices randomly and can write the SVD of this as

$$\mathbf{H}_{\text{eff}}^{[i,j]} = [\mathbf{U}^{[i,j]\,(1)} \quad \mathbf{U}^{[i,j]\,(0)}][\mathbf{\Lambda}^{[\mathbf{i},\mathbf{j}]} \quad \mathbf{O}]^T\mathbf{X}^{[i,j]\,H} \tag{10}$$

where the matrix $\mathbf{U}^{[i,j]\,(0)}$ is composed of the last $\frac{M}{2} \times 1$ left singular vectors. Then from (10) the receiver beamforming can be performed as $\bar{\mathbf{M}}_{\text{zf}}^{[i]} = \mathbf{U}^{[i,j]\,(0)\,H}$. Hence after receive beamforming the interference is nullified and only desired signal $\bar{\mathbf{M}}_{\text{zf}}^{[i]\,H}\mathbf{H}_{\text{eff}}^{[i,i]}\mathbf{C}_{\text{zf}}^{[l]}\mathbf{s}^{[i]}$ remains. The authors of [2] then used the information rate maximization criteria to find the optimal values of $\bar{\mathbf{M}}_{\text{zf}}^{[i]}$ and $\mathbf{V}_{\text{zf}}^{[i]}$ as

$$\bar{\mathbf{M}}_{\text{zf}}^{[i]} = \bar{\mathbf{M}}_{\text{zf}}^{[i]} \mathbf{U}_{\text{zf}}^{[i]\,H} \quad (11)$$

$$\mathbf{V}_{\text{zf}}^{[i]} = \mathbf{Q}^{[i]} \mathbf{X}_{\text{zf}}^{[i]} \mathbf{\Sigma}^{[i]\,\frac{1}{2}} \quad (12)$$

where $\mathbf{\Sigma}^{[i]}$ is water-filling matrix [2] and satisfy the power constraint as $\text{Tr}(\mathbf{\Sigma}^{[i]}) \leq P$.

So till now we have seen the closed form solution for $L \leq 3$ and the precoder optimization taking care of sum rate maximization. In next section we will briefly discuss the iterative algorithms to achieve IA when $L \geq 3$.

IV. ITERATIVE ALGORITHMS

In this section we will change our channel model as $\mathbf{H}^{[i,j]} \in \mathbb{C}^{N \times M}$ i.e. we will consider M antennas at the BS and N antennas at the receiver. The receiver beamforming matrices will now be $\mathbf{U}^{[k]}$.

A. Feasibility of Alignment

Before we go into the iterative algorithms we need to discuss the feasibility criteria for dof allocation. Given the channel matrices $H^{[k,j]} \quad k,j \in K$, we say that the dof allocation $(d^{[1]}, d^{[2]}, ..., d^{[k]})$ is feasible if there exist transmit precoding matrices $\mathbf{V}^{[k]}$ and receive interference suppression matrices $\mathbf{U}^{[k]}$

$$\begin{aligned} \mathbf{V}^{[k]} &: M^{[k]} \times d^{[k]}, \quad \mathbf{V}^{[k]\,H}\mathbf{V}^{[k]} = \mathbf{I}_{d^{[k]}} \\ \mathbf{U}^{[k]} &: N^{[k]} \times d^{[k]}, \quad \mathbf{U}^{[k]\,H}\mathbf{U}^{[k]} = \mathbf{I}_{d^{[k]}} \end{aligned} \quad (13)$$

such that

$$\begin{aligned} \mathbf{U}^{[k]\,H}\mathbf{H}^{[k,j]}\mathbf{V}^{[j]} &= 0, \forall j \neq k \quad (14) \\ \text{rank}(\mathbf{U}^{[k]\,H}\mathbf{H}^{[k,k]}\mathbf{V}^{[k]}) &= d^{[k]}, \forall k \quad (15) \end{aligned}$$

B. Reciprocity of Alignment

An interesting observation from the problem formulation above is the duality relationship between interference alignment on a given interference channel and its reciprocal channel obtained by switching the direction of communication. Specifically, let $\overleftarrow{\mathbf{V}}^{[k]}, \overleftarrow{\mathbf{U}}^{[k]}$ denote the transmit precoding filters and the receive interference suppression filters on the reciprocal channel. The feasibility conditions on the reciprocal channel are:

$$\begin{aligned} \overleftarrow{\mathbf{V}}^{[k]} &: N^{[k]} \times d^{[k]}, \quad \overleftarrow{\mathbf{V}}^{[k]\,H}\overleftarrow{\mathbf{V}}^{[k]} = \mathbf{I}_{d^{[k]}} \\ \overleftarrow{\mathbf{U}}^{[k]} &: M^{[k]} \times d^{[k]}, \quad \overleftarrow{\mathbf{U}}^{[k]\,H}\overleftarrow{\mathbf{U}}^{[k]} = \mathbf{I}_{d^{[k]}} \end{aligned} \quad (16)$$

such that

$$\begin{aligned} \overleftarrow{\mathbf{U}}^{[k]\,H}\,\mathbf{H}^{[k,j]}\,\overleftarrow{\mathbf{V}}^{[j]} &= 0, \forall j \neq k \quad (17) \\ \text{rank}(\overleftarrow{\mathbf{U}}^{[k]\,H}\,\mathbf{H}^{[k,k]}\,\overleftarrow{\mathbf{V}}^{[k]}) &= d^{[k]}, \forall k \quad (18) \end{aligned}$$

Suppose we set $\overleftarrow{\mathbf{V}}^{[k]} = \mathbf{U}^{[k]}, \overleftarrow{\mathbf{U}}^{[k]} = \mathbf{V}^{[k]}$. Then the feasibility Conditions on the reciprocal channel become identical to the original feasibility conditions. Thus, the following observation can be made:

Reciprocity of Alignment: Since the feasibility conditions are identical, if the degrees of freedom allocation $(d^{[1]}, d^{[2]}, ..., d^{[k]})$ is feasible on the original interference network then it is also feasible on the reciprocal network (and vice versa). Interference alignment on the reciprocal interference network is simply achieved by choosing the transmit filters and the receive filters on the reciprocal channel as the receive filters and the transmit filters (respectively) of the original channel.

Reciprocity of alignment is a key property used for distributed interference alignment algorithms, described in the next section.

C. DISTRIBUTED ALGORITHM FOR INTERFERENCE ALIGNMENT

In this section we construct distributed interference alignment algorithms for the interference channel with multiple antenna nodes and no symbol extensions.

Since we are interested in distributed algorithms, we start with arbitrary transmit and receive filters $\mathbf{U}^{[k]}, \mathbf{V}^{[k]}$ and iteratively update these filters to approach interference alignment. The quality of alignment is measured by the power in the leakage interference at each receiver, i.e. the interference power remaining in the received signal after the receive interference suppression filter is applied. The goal is to achieve interference alignment by progressively reducing the leakage interference. If interference alignment is feasible then eventually leakage interference will be zero. The iterative procedure is as follows:

Iterative Interference Alignment

1: Start with arbitrary precoding matrices $\mathbf{V}^{[j]} : M^{[j]} \times d^{[j]}, \mathbf{V}^{[j]} \times \mathbf{V}^{[j]\,H} = \mathbf{I}_{d^{[j]}}$.

2: Begin iteration.

3: Compute interference covariance matrix at the receivers:

$$\mathbf{Q}^{[k]} = \sum_{j=1, j\neq k}^{L} \frac{P^{[j]}}{d^{[j]}} \mathbf{H}^{[k,j]}\mathbf{V}^{[j]}\mathbf{V}^{[j]\,H}\mathbf{H}^{[k,j]\,H}$$

4: Compute the interference suppression matrix at each receiver [3]

$$\mathbf{U}_{*d}^{[k]} = v_d[\mathbf{Q}^{[k]}], \qquad d = 1, ..., d^{[k]}$$

5: Reverse the communication direction and set $\overleftarrow{\mathbf{V}}^{[k]} = \mathbf{U}^{[k]}$.

6: Compute interference covariance matrix at the new receivers:

$$\overleftarrow{\mathbf{Q}}^{[k]} = \sum_{k=1, k\neq j}^{L} \frac{\overleftarrow{P}^{[k]}}{d^{[k]}} \overleftarrow{\mathbf{H}}^{[j,k]}\overleftarrow{\mathbf{V}}^{[k]}\overleftarrow{\mathbf{V}}^{[k]\,H}\overleftarrow{\mathbf{H}}^{[j,k]\,H}$$

7: Compute the interference suppression matrix at each receiver:

$$\overleftarrow{\mathbf{U}}_{*d}^{[j]} = v_d[\overleftarrow{\mathbf{Q}}^{[j]}], \qquad d = 1, ..., d^{[k]}$$

8: Reverse the communication direction and set $\mathbf{V}^{[k]} = \overleftarrow{\mathbf{U}}^{[k]}$.

9: Continue till convergence.

In the algorithm written above, the notation used is dth column of $\mathbf{U}^{[k]}$ is given by $\mathbf{U}^{[k]}_{*d}$ and $v_d[\mathbf{A}]$ is the eigenvector corresponding to the dth smallest eigenvalue of $\mathbf{A}$.

The following observations summarize the intuition behind the iterative algorithm.

1) Dimensions along which a receiver sees the least interference from other nodes are also the dimensions along which it causes the least interference to other nodes in the reciprocal network where it functions as a transmitter.
2) The weighted leakage interference is unchanged in the original and reciprocal networks if the transmit and receive filters are switched.

D. Maximum SINR algorithm

The algorithm presented above seeks perfect interference alignment. However, note that interference alignment makes no attempt to maximize the desired signal power within the desired signal subspace. In fact the algorithm described above does not depend at all on the direct channels $\mathbf{H}^{[kk]}$ through which the desired signal arrives at the intended receiver. Therefore, while the interference is eliminated within the desired space, no coherent combining gain (array gain) for the desired signal is obtained with interference alignment. While this is optimal as all signal powers approach infinity, it is not optimal in general at intermediate SNR values.

We consider one such natural extension of the interference alignment algorithm where the receive filters $\mathbf{U}^{[k]}$ and $\overleftarrow{\mathbf{U}}^{[k]}$ are chosen to maximize SINR at the receivers instead of only minimizing the leakage interference. While there is no loss of generality in assuming orthogonal precoding vectors for the streams sent from the same transmitter as far as interference alignment is concerned, orthogonal precoding vectors are in general suboptimal for SINR optimization. We therefore no longer assume that the columns of $\mathbf{V}^{[k]}$ (the transmit precoding vectors) are mutually orthogonal. We also identify the columns of $\mathbf{U}^{[k]}$ to be the specific combining vectors for the corresponding desired data stream, so that they are not necessarily orthogonal either. With these modified definitions, the SINR of the lth stream of the kth receiver is

$$\text{SINR}_{k,l} = \frac{\mathbf{U}^{[k]\,H}_{*l}\mathbf{H}^{[k,k]}\mathbf{V}^{[k]}_{*l}\mathbf{V}^{[k]\,H}_{*l}\mathbf{H}^{[k,k]\,H}\mathbf{U}^{[k]}_{*l}}{\mathbf{U}^{[k]\,H}_{*l}\mathbf{B}^{[k,l]}\mathbf{U}^{[k]}_{*l}} \tag{19}$$

where

$$\mathbf{B}^{[k,l]} = \sum_{j=1}^{L}\frac{P^{[j]}}{d^{[j]}}\sum_{d=1}^{d^{[j]}}\mathbf{H}^{[k,j]}\mathbf{V}^{[j]}_{*d}\mathbf{V}^{[j]\,H}_{*d}\mathbf{H}^{[k,j]\,H} - \frac{P^{[k]}}{d^{[k]}}\mathbf{H}^{[k,k]}\mathbf{V}^{[k]}_{*l}\mathbf{V}^{[k]\,H}_{*l}\mathbf{H}^{[k,k]\,H} \tag{20}$$

The unit vector $\mathbf{U}^{[k]\,H}_{*l}$ that maximizes $\text{SINR}_{k,l}$ is given by

$$\mathbf{U}^{[k]\,H}_{*l} = \frac{(\mathbf{B}^{[k,l]})^{-1}\mathbf{H}^{[k,k]}\mathbf{V}^{[k]}_{*l}}{||(\mathbf{B}^{[k,l]})^{-1}\mathbf{H}^{[k,k]}\mathbf{V}^{[k]}_{*l}||} \tag{21}$$

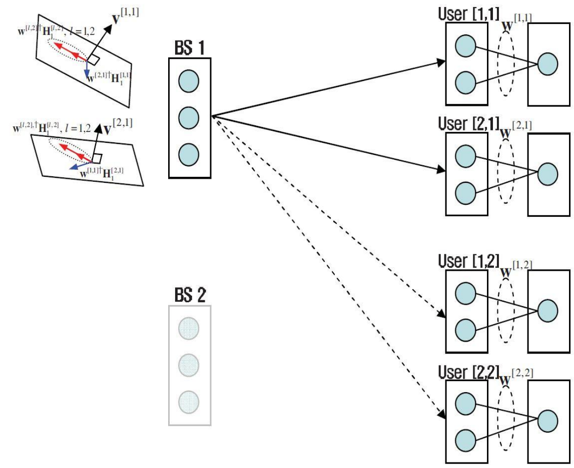

Fig. 2. MIMO-IFBC with grouping for the (3,2,2)

The steps of the algorithm are:

Max SINR Algorithm

1: Start with any $\mathbf{V}^{[k]} : M^{[k]} \times d^{[k]}$, columns of $\mathbf{V}^{[k]}$ are linearly independent unit vectors.
2: Begin iteration.
3: Compute interference plus noise covariance matrix for $\mathbf{B}^{[kl]}$ for stream l at receiver k according to (19), $\forall k \in \{1, 2, ..., L\}, l \in \{1, 2, ..., d^{[k]}\}$.
4: : Calculate receive combining vectors $\mathbf{U}^{[k]}_{*l}$ at receiver k according to (21), $\forall k \in \{1, 2, ..., L\}, l \in \{1, 2, ..., d^{[k]}\}$.
5: Reverse the communication direction and use the receive combining vectors as precoding vectors:$\overleftarrow{\mathbf{V}}^{[k]}= \mathbf{U}^{[k]}$.
6: In the reciprocal network, compute interference plus-noise covariance matrix $\overleftarrow{\mathbf{B}}^{[kl]}$ for stream l at receiver k, $\forall k \in \{1, 2, ..., L\}, l \in \{1, 2, ..., d^{[k]}\}$.
7: Calculate receive combining $\overleftarrow{\mathbf{U}}^{[k]}_{*l}, \forall k \in \{1, 2, ..., L\}, l \in \{1, 2, ..., d^{[k]}\}$.
8: Reverse the communication direction and use the receive combining vectors as precoding vectors: $\mathbf{V}^{[k]} = \overleftarrow{\mathbf{U}}^{[k]}$, $\forall k \in \{1, 2, ..., L\}$.
9: Repeat until convergence.

V. MIMO IFBC SYSTEMS

As we know, in IFBC system multiple users are supported in each BS, and the users in each BS to be supported is K. The notation used for user$-i$ in cell$-j$ is $[i, j]$. In this section an explicit IA scheme is shown, which mitigates both ICI and IUI simultaneously in the two-cell two-user MIMO-IFBC, and iterative computation is also not required.

To explain this, we start with a simple case of $(M, N, K) = (3, 2, 2)$ as shown in Fig. 2. The BS 1 wants to deliver two symbols, $\mathbf{s}^{[1,1]}$ and $\mathbf{s}^{[2,1]}$, to the user $[1, 1]$ and user $[2, 1]$ using the transmit beamforming vectors $\mathbf{v}^{[1,1]}$ and $\mathbf{v}^{[2,1]}$, respectively. In general, for given receive beamforming vectors, the minimum number of transmit antennas is 4 so that the transmit beamforming vectors cancel out all ICI and IUI but this scheme achieves the cancellation of both ICI and IUI with 3 transmit antennas. In the following two steps transmit and receive beamforming method enabling the ICI channel alignment is presented.

Step 1: Designing the receive beamforming vectors The user $[1,2]$ and user $[2,2]$ design the receive beamforming vectors $\mathbf{w}^{[1,2]}$ and $\mathbf{w}^{[2,2]}$, respectively so that the ICI channel are aligned with each other, which is

$$\text{span}(\mathbf{H}_1^{[1,2]\,H}\mathbf{U}^{[1,2]}) = \text{span}(\mathbf{H}_1^{[2,2]\,H}\mathbf{U}^{[2,2]}) \tag{22}$$

The intersection subspace can be computed by solving the following matrix equation,

$$\begin{bmatrix} \mathbf{I}_M & -\mathbf{H}^{[1,2]\,H} & \mathbf{0} \\ \mathbf{I}_M & \mathbf{0} & -\mathbf{H}^{[2,2]\,H} \end{bmatrix} \begin{bmatrix} \mathbf{h}_2^{\mathcal{ICI}} \\ \mathbf{U}^{[1,2]} \\ \mathbf{U}^{[2,2]} \end{bmatrix} = \mathbf{M}_1\mathbf{x}_1 = \mathbf{0} \tag{23}$$

where $\mathbf{h}_2^{\mathcal{ICI}}$ implies the direction of aligned effective interference channels from the BS 1 to the user $[1,2]$ and user $[2,2]$ after applying the receiver beamforming vectors. Since the size of the matrix $\mathbf{M}_1$ is 6×7, it has one dimensional null space. Therefore, the receive beamforming vectors for ICI channel alignment can be obtained explicitly with probability one.

Step 2: Choosing the transmit beamforming vectors Since the effective ICI channels are aligned with each other, the BS 1 can consider two different ICI channel vectors as a one ICI channel vector which spans one dimensional subspace as shown in Fig. 2. Therefore, the beamforming vectors $\mathbf{v}^{[1,1]}$ and $\mathbf{v}^{[2,]}$ are designed as,

$$\begin{aligned} \mathbf{v}^{[1,1]} &\subset \text{null}\left(\left[(\mathbf{U}^{[2,1]\,H}\mathbf{H}_1^{[2,1]})^H \quad \mathbf{h}_2^{\mathcal{ICI}}\right]\right) \\ \mathbf{v}^{[2,1]} &\subset \text{null}\left(\left[(\mathbf{U}^{[1,1]\,H}\mathbf{H}_1^{[1,1]})^H \quad \mathbf{h}_2^{\mathcal{ICI}}\right]\right) \end{aligned} \tag{24}$$

Remark 1: In the two-cell MIMO-IFBC where two users are active for each cell, and each BS and user are equipped with 3 transmit antennas and 2 receive antennas, respectively, we can design an IA scheme achieving dof 4 while the time division multiplexing (TDM) can achieve dof 3 [4].

General case for multiple receive antenna $N > 2$ We will now write the generalize version of IA scheme [4] explained in previous section for multiple receive antennas $N > 2$.

For the two-cell MIMO-IFBC where each BS supports two users simultaneously, we can achieve $2N$ degrees of freedom if $M \geq \lceil \frac{3}{2}N \rceil$ where the BS and user have M and N antennas, respectively.

Remark 2: We can expect that the degrees of freedom of the two-cell $(M,N,2)$ MIMO-IFBC cannot be larger than that with user cooperation between two users of each cell as an trivial outerbound. Interestingly, the two-cell $(M,N,2)$ MIMO-IFBC with user cooperation can be regarded as a two-user MIMO-IFC when M transmit antennas and $2N$ receive antenna are employed, and the degrees of freedom is characterized as follows

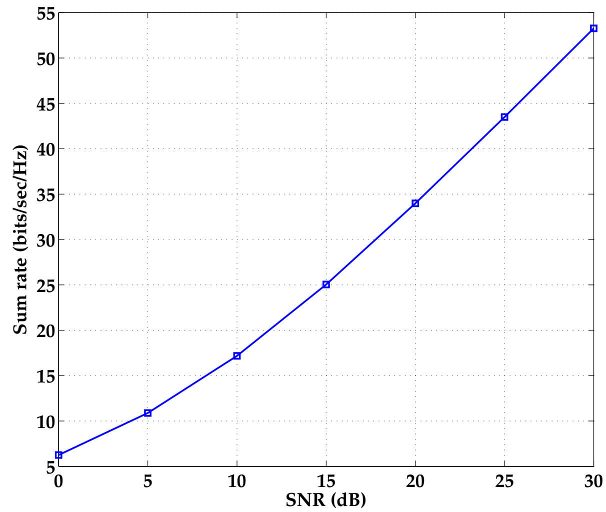

Fig. 3. Sum rate versus SNR for $M = 4$ using precoding optimization with zero-forcing receiver

$$d^{[1,1]} + d^{[2,1]} + d^{[1,2]} + d^{[2,2]} \leq \min\{2M, 4N, \max(M, 2N)\} \tag{25}$$

When $\lceil \frac{3}{2} \rceil N \leq M < 2N$, the outerbound on the dof can be obtained as $2N$, which coincides with the achievable degrees of freedom of the aforementioned interference alignment scheme and this result implies that the proposed interference alignment scheme can achieve optimal degrees of freedom for the two-cell $(M,N,2)$ MIMO-IFBC.

VI. Extended Grouping Scheme

In previous section we saw that the authors of [4] have proposed a grouping method to achieve IA optimally. However they proposed this scheme for only two cell scenario. Therefore, to generalize this result, the authors of [5] have provided a extended grouping method.

The equations for grouping are very much similar to the ones we have seen in previous section, but the main difference is which we are addressing. In previous section we had only two cells and hence we were grouping the users in the neighbouring cell. But since in extended scheme we have more than two cells we have to cyclically group the users. For e.g. the BS 1 will group the users in cell 2, BS 2 will group the users in cell 3 and hence the BS L will group the users of cell 1. The order of this cycle can be reversed which is not going to affect the grouping scheme.

We will not show the detailed analysis of this extended scheme as it is much similar but we will show the sum rate in simulations to appreciate the IFBC systems.

VII. Simulation Results

In this section we will reproduce the simulation results for some of the schemes discussed.

In Fig. 3 we have shown the plot of sum rate versus SNR (dB) for precoder optimization using zero forcing receiver as we have discussed in section-III.

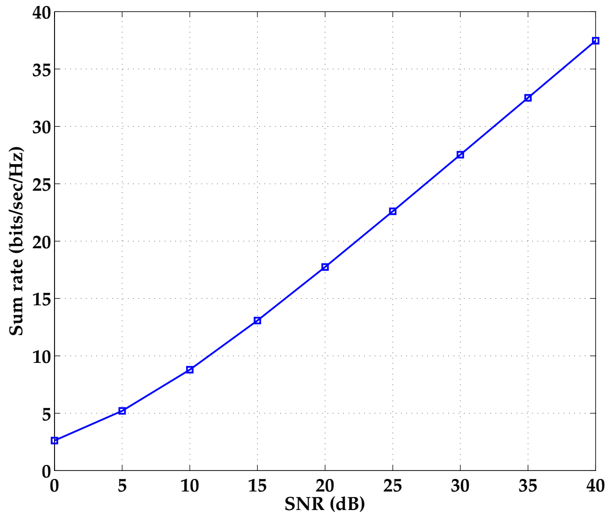

Fig. 4. Performance of distributed algorithm when $M = 2$, $L = 3$

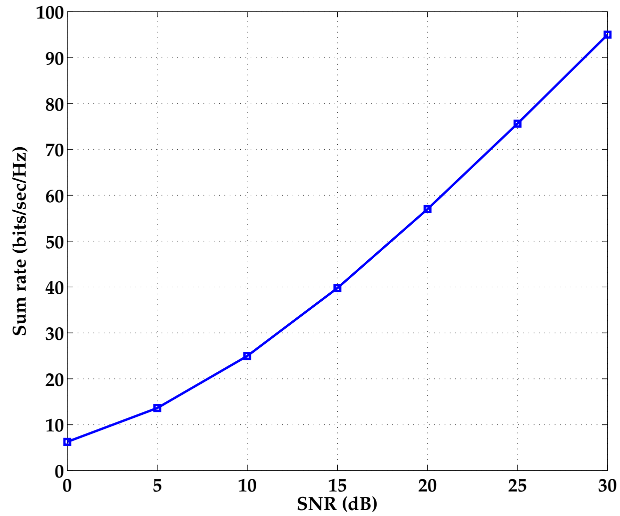

Fig. 6. Sum rate versus SNR for extended grouping scheme when $(M, N, L, K, d) = (10, 6, 3, 2, 2)$ MIMO-IFBC.

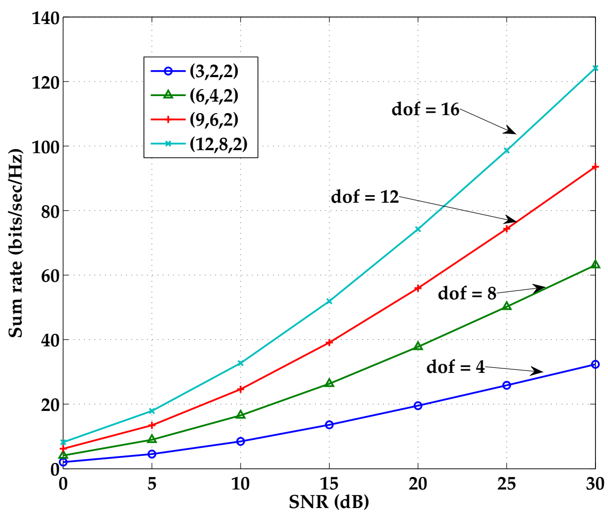

Fig. 5. Achievable degrees of freedom with various M and N parameters for $(M, N, 2)$ MIMO-IFBC.

In Fig. 4 we have shown the plot of sum rate versus SNR (dB) when $M = 2$ and $L = 3$ for distributed algorithm to achieve IA discussed in section-IV.

In Fig. 5 we have shown the sum rate achieved versus SNR in dB for the grouping method discussed in section-V. The results are as expected, as the slope of the sum rate curve comes out to be the dof achieved by the scheme.

In Fig. 6 we have shown the sum rate achieved versus SNR in dB for the extended grouping method discussed in section-VI. The results are as expected, as the slope of the sum rate curve comes out to be the dof achieved by the scheme.

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