# User selection in MIMO Interfering Broadcast Channels

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# Process flow

- Background
  - Interference Alignment (IA)
- System Model
  - Achievability in IFBC
- User selection
  - Diversity gain in Interference models
  - User selection problem
  - Coordinate Ascent Approach
  - User selection algorithms
    - IFBC
- Conclusion

# Interference Alignment

- Interference Alignment [1] is a promising technique to achieve maximum degrees-of-freedom in interference systems

- degrees-of-freedom (dof)
  - can be looked upon as interference free dimensions available for signal transmission and reception
  - Key parameter in multi-dimension transmission techniques as the dof serves as pre-log factor in channel capacity

$$C(P,d) = d \times \log_2\left(1 + \frac{P}{\sigma^2}\right)$$ ,where $d$ is dof

[1] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," IEEE Trans. Inf. Theory, vol. 54, pp. 3425–3441, Aug. 2008.

# Interference Alignment (contd.)

- Technique: The received interference is aligned to half the signal dimension and rest half is dedicated for desired signal
- This way for $N$-user interference symmetric channel ($N$ transmitter as well as receiver with same antenna configuration) the achievable dof becomes $\frac{N}{2}$
- Since desired signal and aligned interference spans different dimensions, the desired signal can easily be recovered using orthogonal beamforming at the receiver

# System model

- We will assume that number of antennas at the base station (M) is always greater than equal to that at the receiver (N) i.e. $M \geq N$
- The received signal can be written as

$$y_k^{[l]} = \sum_{j=1}^{L} H_k^{[l,j]} \sum_{i=1}^{K} x_i^{[j]}$$

$$= H_k^{[l,l]} V_k^{[l]} s_k^{[l]} + \sum_{i=0, i \neq k}^{K} H_k^{[l,l]} V_i^{[l]} s_i^{[l]} + \sum_{j=1, j \neq l}^{L} \sum_{i=1}^{K} H_k^{[l,j]} V_i^{[j]} s_i^{[j]} + n_k^{[l]}$$

where,

$H_k^{[l,j]}$ is the channel matrix from the $j$th Base station to the $k$th user in the $l$th cell, $n_k^{[l]}$ is AWGN at the receiver and $x_k^{[l]}$ is defined as

# System model (contd.)

$$x_k^{[l]} = \sum_{i=1}^{d} v_{k,i}^{[l]} s_{k,i}^{[l]} = V_k^{[l]} s_k^{[l]}$$

in which $s_{k,i}^{[l]}$ is the $i$th symbol precoded using linear beamforming vector $v_k^{[l]}$

- The received signal after receiver beamforming is written as

$$\tilde{y}_k^{[l]} = U_k^{[l]H} H_{k_L}^{[l,l]} V_k^{[l]} \underset{K}{s}{}_k^{[l]} +$$

$$U_k^{[l]H} \Big( \sum_{i=1, i \neq k}^{K} H_k^{[l,l]} V_i^{[l]} s_i^{[l]} + \sum_{j=1, j \neq l} \sum_{i=1} H_k^{[l,j]} V_i^{[j]} s_i^{[j]} \Big) + \tilde{n}_k^{[l]}$$

where, $U_k^{[l]}$ is the receive beamforming matrix and $\tilde{n}$ is the effective noise

# Achievability in IFBC

- A solution which we refer as grouping solution [2] exist for a special case of 2-cell and is known to achieve dof upper-bound
- The solution works on the principle of grouping the neighboring cell's users to reduce the effective dimension of ICI while designing beamformer
- This reduction in effective dimension in turn reduces minimum required antennas and hence achieve higher dof for given $M$ and $N$
- This method achieves same dof as we can achieve on full user cooperation (upperbound)

[2] W. Shin, N. Lee, J.-B. Lim, C. Shin, and K. Jang, "On the design of interference alignment scheme for two-cell MIMO interfering broadcast channels," IEEE Trans. Wireless Commun., vol. 10, no. 2, pp. 437–442, Feb. 2011.

# Achievability in IFBC (contd.)

- This solution will work only for 2-cell system and hence a more generalized solution [3] exist working on the same principle of grouping
- The users are grouped cyclically i.e. if there are 5 cells then, for designing beamformers of 1 users in cell-2 will be grouped, for 2 users in cell-3 and for 5 users in cell-1 will be grouped

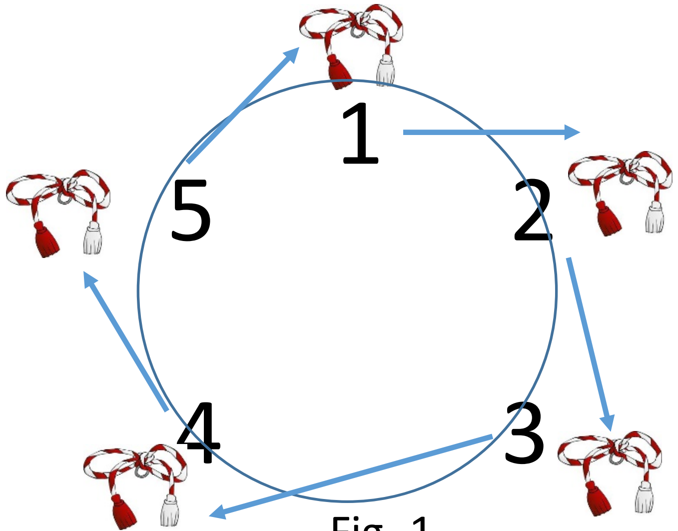


Fig. 1

[3] J. Tang and S. Lambotharan, "Interference alignment techniques for MIMO multi-cell interfering broadcast channels," IEEE Trans. Commun., vol. 61, no. 1, pp. 164–175, Jan. 2013.

# Achievability in IFBC (contd.)

- The minimum required antennas for extended grouping scheme can be written as

$$M \geq [K(L-1)+1] \times d$$

$$N \geq [(K-1)(L-1)+1] \times d$$

# User selection

## Multiuser Diversity

- Exploitation of the fact that probability of finding the user with better channel increases as we increase our search range
- Two possible approach when we have to select specified number of users among available
    1. Random user selection
    2. User selection by performing search (usually following some criteria)
- First process is easier to implement as it is just picking a user (or users) randomly while second one involves some computation which indeed depends upon search method and criteria of search

# Multiuser diversity (contd.)

Lets say we have 5 users with the following channel values and transmit power 10dB

$h_1 = 0.1, h_2 = 1, h_3 = 0, h_4 = 0.5$ and $h_5 = 1.5$

The corresponding channel capacity ($\log_2(1 + P|h|^2)$) will be

$C_1 = 0.137, C_2 = 3.45, C_3 = 0, C_4 = 0.5$ and $C_5 = 4.554$

- If we perform a user selection based on sum-rate maximization, then our obvious choice would be user-5 and our achievable sum-rate will be $4.554$ bits/s/Hz
- On the other hand if we perform random user selection then our achievable sum rate will be (assume every user is equally likely to get selected)

$C = \frac{1}{5} \times (0.137 + 3.45 + 0 + 0.5 + 4.554) = 1.728$ bits/s/Hz

# User selection problem

- The problem of user selection uses a selection criteria and a given constraint
- The selection criteria could be sum-rate of the system, Bit-Error-Rate etc.
- The constraint is usually the resources like Power, antennas etc.
- We will talk about performing user selection for maximization of sum-rate of the system under given power constraint
- The easiest way of formulating user selection is by

→ performing a search over all possible user-subsets among available → compute sum-rate of each user-subset

→ select the subset having maximum sum-rate

# User selection problem (contd.)

- The problem of user selection with exhaustive search can then be written in mathematical form as

$$R_{opt} = \max_{S^{[l]} \subset \Gamma, \left|S^{[l]}\right| = K, \forall l} R\left(S^{[1]}, S^{[2]}, \ldots, S^{[L]}\right)$$

  where, $S^{[l]}$ is the subset of users selected in the $l$th cell, $\Gamma$ is the set of total users in each cell and $K$ is the number of users selected in each cell. Hence for IFBC $K > 1$

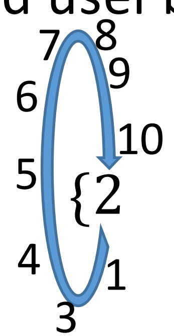
- The solution obtained using brute-force approach will be termed as optimal solution
- The computational complexity of brute-force search is huge making it impractical to run. This sets the need for less complex user selection algorithms which have good achievable sum-rate and is currently an active area of research

# User selection

## Coordinate Ascent Approach

- In coordinate ascent approach [4] we will initialize the user subset based on some criteria (usually channel energy) and then iterate each user index while keeping other ones constant
- For example we have $10$ available users ($|\Gamma| = 10$) in each cell ($N = 3$) and we have initialized our user subset as $G = \{2,3,6\}$ i.e. we have selected user-2 in cell-1, user-3 in cell-2 and user-6 in cell-3 in the initialization step
- We will then iterate each selected user based on some criteria (sum-rate here) as

$$G_{next} = \{2 \quad ,3,6\}$$




This step will get repeated for user in cell-2 and cell-3


[4] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*. Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1997.

# User selection in IFBC

User selection in IFBC:

- Multiple users are selected in each cell ($K > 1$)
- Two low complexity user selection algorithms for IFBC were introduced in [5]
- The algorithms use extended grouping scheme [3] for designing transmit and receiver beamformer
- It is shown that both these algorithms have linear computational complexity as compared to exponential of the brute-force approach
- The sum-rate achieved by these algorithms is also shown to be close to the optimal solution

[5] G. Gupta and A.K. Chaturvedi, “User Selection in MIMO Interfering Broadcast Channels,” IEEE Trans. Commun., vol. 62, no. 5, pp 1568-1576, May 2014.

# User selection in IFBC (contd.)

## Algorithm-1 (s-algorithm)

- The algorithm uses coordinate ascent approach to select the users and hence the same procedure will follow as we have seen except for multiple user selection in each cell
- However, to avoid unnecessary computation of receive and transmit beamformer, the algorithm identifies the identical computations and avoid it
  - For example, while varying user index in a particular cell, $U$ and $V$ of the users in other cells could be reused
- The sum-rate is used as the criteria for prioritizing the users while performing secondary search

# User selection in IFBC (contd.)

## Algorithm-2 (o-algorithm)

- The algorithm-1 is able to reduce the search range (and hence complexity) using coordinate ascent approach but computation of sum-rate at each step is still expensive
- To avoid the computation of beamformers at each step some more insight to grouping scheme has to be developed
- The problem of user selection in IFBC is complicated by the fact that a user in a particular cell is effected by the remaining users in its own cell as well as that in rest of the cells in the system
- The basic idea behind grouping scheme is to group the users in the neighboring cell in order to reduce the effective dimension of the ICI

# User selection in IFBC (contd.)

- Mathematically, it can be written as

$$G = span\left\{H_1^{[l+1,l]\,H} U_1^{[l+1]}\right\} = span\left\{H_2^{[l+1,l]\,H} U_2^{[l+1]}\right\} = \cdots$$
$$= span\left\{H_K^{[l+1,l]\,H} U_K^{[l+1]}\right\}$$

where, $H_k^{[i,j]}$ is channel from $j$th transmitter to $k$th user in the $i$th cell and $U_k^{[i]}$ is the receive beamformer of the $k$th user in the $i$th cell

- We can avoid the computation of transmit beamformer if we express the effective downlink channel at the receiver without it and take care of grouping with above equation
- The effective downlink channel at receiver is of the form of $HV$ ($V$ is transmit beamformer) but we need it to take the form of $H^H U$ to consider the effects of grouping

# User selection in IFBC (contd.)

- We will use the concept of reciprocal channel to do this transformation but before that lets define reciprocal model [6], [7]

Reciprocal channel model:

- The transmitter will become receiver and the receiver will become transmitter in the reciprocal system
- The transmit power constraint will remain same in the reciprocal channel
- The channel in the original system ($H$) will become $\overleftarrow{H} = H^H$ in the reciprocal system


[6] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," in *Proc. 2008 IEEE GLOBECOM.*

[7] B. Babadi and V. Tarokh, "A distributed dynamic frequency allocation algorithm," 2007. [Online]. Available: http://arxiv.org/abs/0711.3247

# User selection in IFBC (contd.)

- There is another useful concept called Reciprocity of Alignment [7] which states that the basic requirements of nulling the ICI, IUI and achieve required dof will not change in the reciprocal channel if we interchange receiver and transmit beamformer

i.e. make $\overleftarrow{V} = U$ and $\overleftarrow{U} = V$

- We can now say that our required effective channel $(H^H U)$ is nothing but $\overleftarrow{H}\overleftarrow{V}$ in the reciprocal channel and hence we can take our entire problem into reciprocal channel model without affecting the attainable dof (reciprocity of alignment)

# User selection in IFBC (contd.)

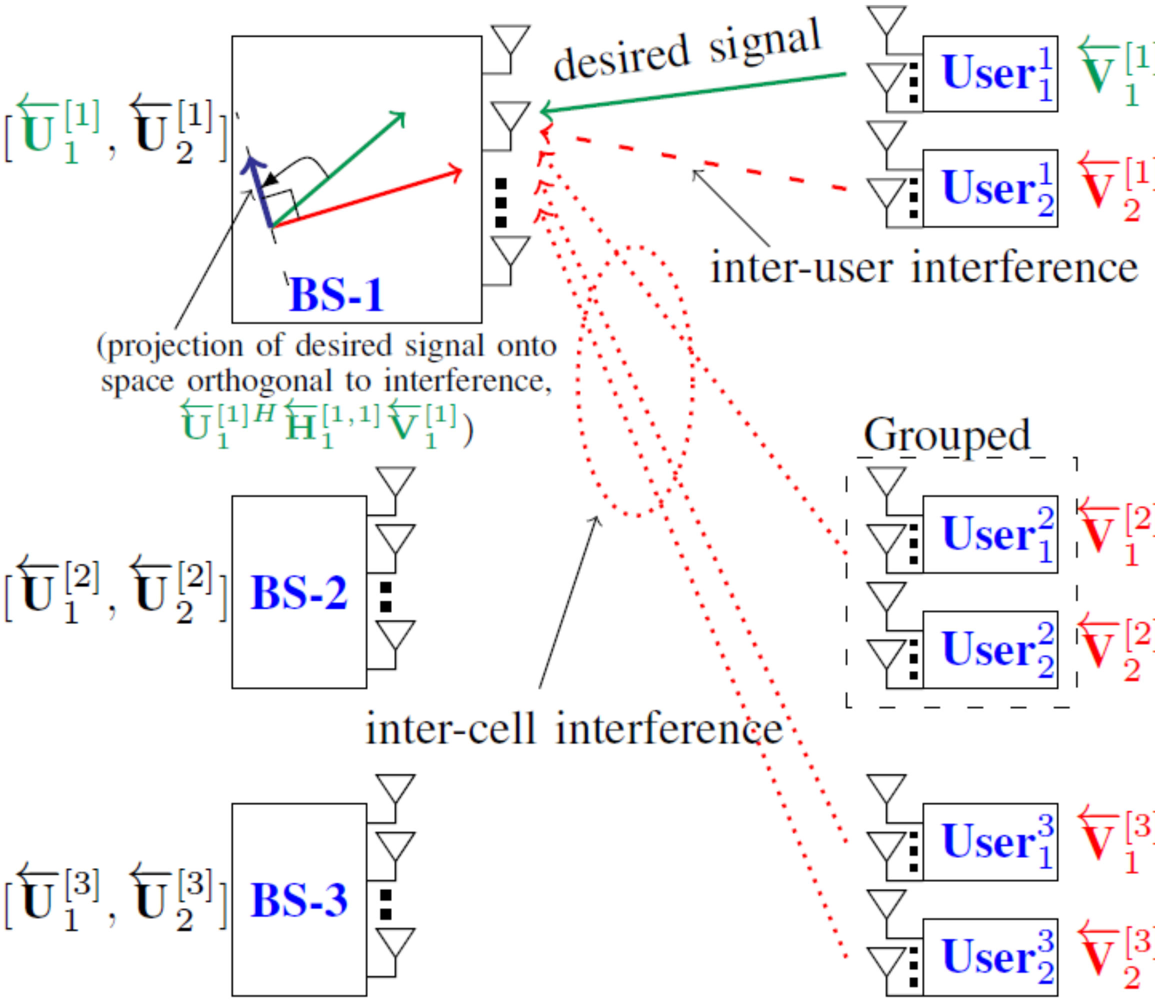



Fig. 2 Reciprocal channel of IFBC

# User selection in IFBC (contd.)

- As clear from the figure that we need to select the user such that the effective interference (ICI + IUI) is more orthogonal to the desired signal in the reciprocal channel
- To account orthogonality quantitatively we will use the concept of chordal distance [8]

Chordal distance:

- *Grassmannian space*: The Grassmannian space $G(m, n)$ is the set of all n-dimensional subspaces of Euclidean m-dimensional space
- *Generator matrix*: A $m \times n$ matrix is called the generator matrix for an n-plane $P \in G(m, n)$ if its columns span $P$.

[8] J. H. Conway, R. H. Hardin, and N. J. A. Sloane, "Packing lines, plane, etc.: packings in grassmannian spaces," Exper. Math, vol. 5, no. 2, pp. 139–159, 1996.

# User selection in IFBC (contd.)

- Suppose $A_G$ and $B_G$ are generator matrices of planes $P$ and $Q$, columns of which are orthonormal vectors, then the chordal distance between $P$ and $Q$ is defined as

$$d(P,Q) = \frac{1}{\sqrt{2}} \left\| A_G A_G^H - B_G B_G^H \right\|_F$$

where, $\|A\|_F$ denote frobenius norm of matrix $A$

- Therefore, the o-algorithm looks for the user which maximize the chordal distance between the effective interference space and the desired signal space in the reciprocal channel

# User selection in IFBC (contd.)

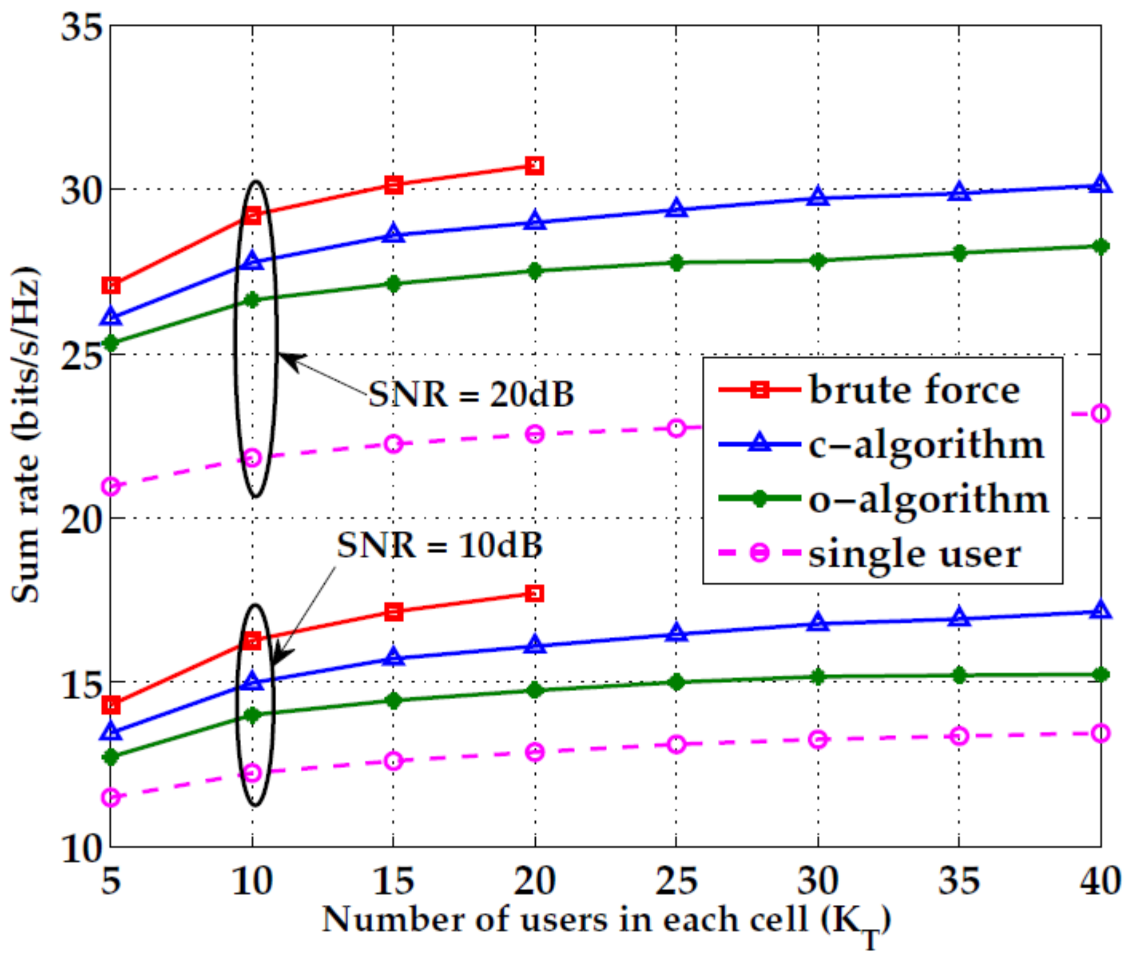



Fig. 3 Sum-rate vs Number of users when $M = 3, N = 2, L = 2, K = 2$ and $d = 1$

# User selection in IFBC (contd.)

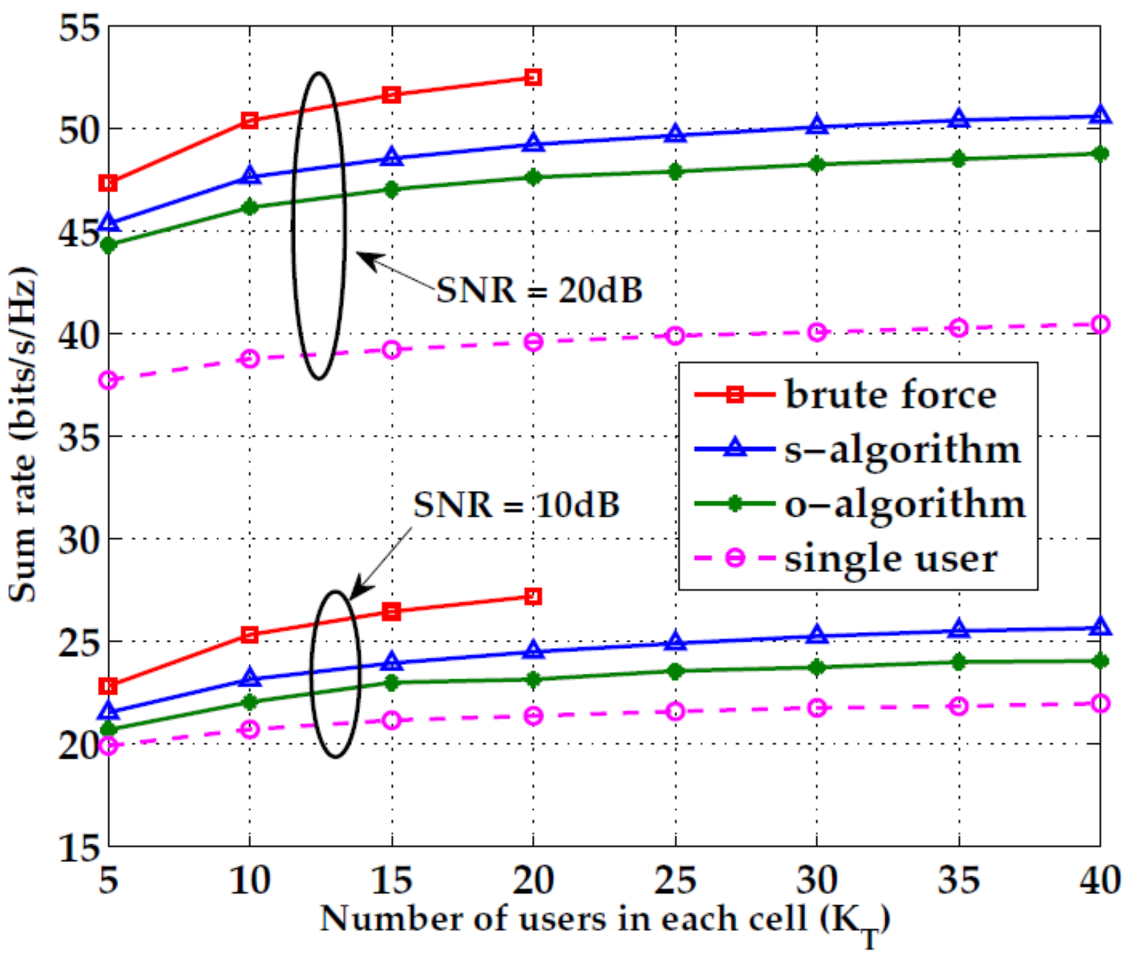



Fig. 4 Sum-rate vs Number of users when $M = 6, N = 4, L = 2, K = 2$ and $d = 2$

# User selection in IFBC (contd.)

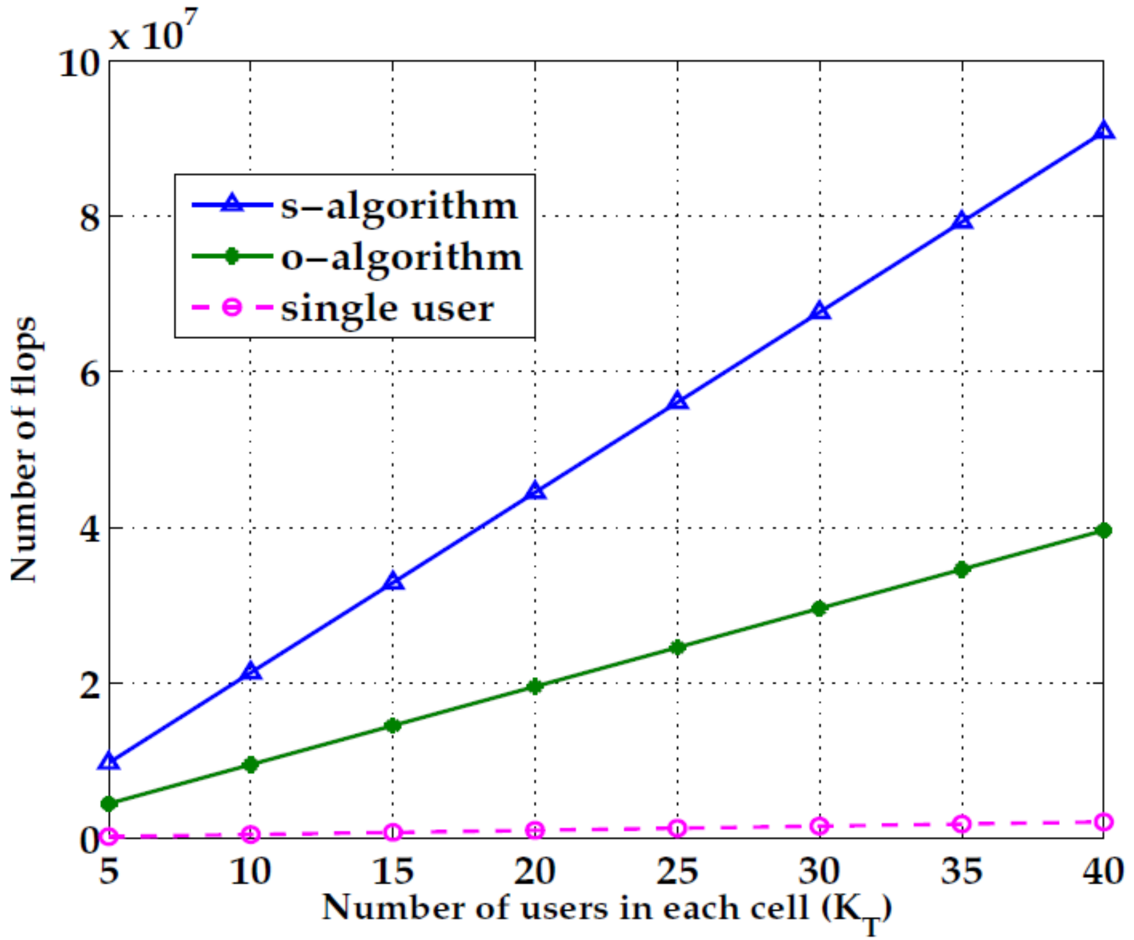



Fig. 5 Number of flops vs Number of users when $M = 6, N = 4, L = 2, K = 2$ and $d = 2$

# Conclusions

- We have seen an achievability scheme for IFBC namely grouping scheme
- Multiuser diversity has been exploited to increase the sum-rate of the system
  - User selection algorithms are employed
- Two novel user selection algorithms for IFBC are developed and their performance is evaluated using Monte-Carlo Simulations
- The algorithms offer significant savings in computational complexity and a transition from exponential order complexity of the Brute-force search to the linear order in 'o' and 's-algorithm'

# Conclusions

- The o-algorithm is better than s-algorithm in terms of computational complexity but has slightly less sum rate performance than s-algorithm, hence there is a trade-off
- Therefore, it can be said that for large number of users in each cell we should go for o-algorithm and for small numbers we should go for s-algorithm